*KINEMATIC GEOMETRY. INVERSION AND INVERSORS.

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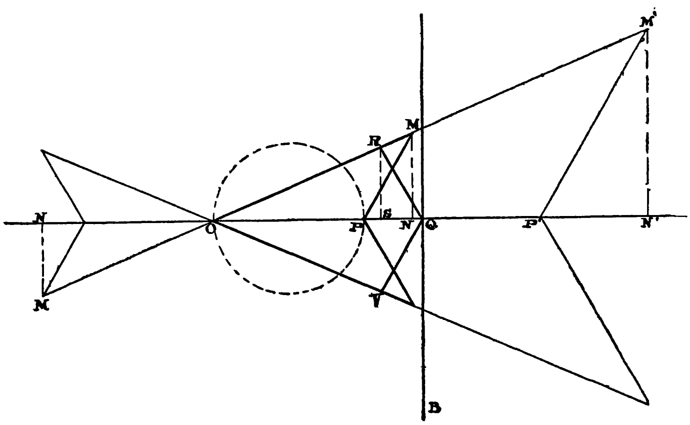
This paper is in the nature of a supplement to the one read by the author before Section A, American Association for the Advancement of Science, in Philadelphia, 1904. In that communication two types of Inversors were exhibited and demonstrated. They represented two distinct groups each embodying the property of inversion, and for special cases they become instruments for describing a line which is mathematically straight.

Further investigation has revealed the fact that an infinite variety of those instruments can be made possessing this principle, differing somewhat in appearance from one another yet essentially the same.

The manner in which they can be constructed is set forth in the following theorems:

THEOREM. *If from any point P in the axis of symmetry OQ of a concave or convex kite lines be drawn parallel to the shorter sides, and terminated by the longer sides (produced if necessary) the product of the distances* $OP \times OQ$ *is constant, whether the point P be taken within or without the points O and Q.*



GIVEN: The kite $ORQV$; OR produced to M; $MP \parallel QV$; MN and RS perpendicular to OQ.

PROOF. $OP = ON \pm PN = \sqrt{(OM^2 - MN^2)} \pm \sqrt{(MP^2 - MN^2)}$,

$OQ = OS \pm QS = \sqrt{(OR^2 - RS^2)} \pm \sqrt{(RQ^2 - RS^2)}$.

Now $RS = \frac{RO}{MO} . MN$; and $RQ = \frac{RO}{MO} . MP$. Substituting we get

$$OP.OQ = \frac{RO}{MO}[OM^2 \pm MP^2], \text{ a constant.}$$

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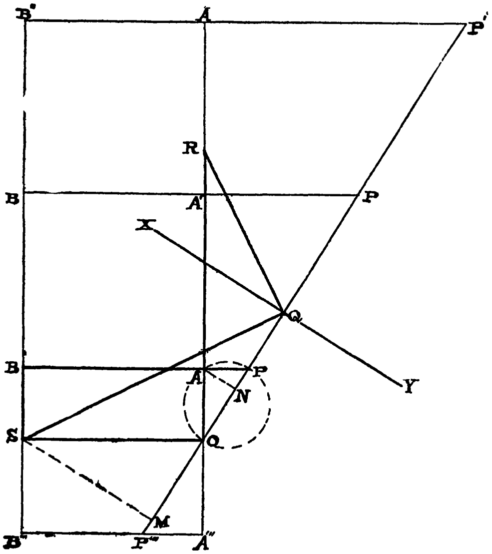
*Read before Section A, American Association for the Advancement of Science, at the New Orleans meeting, December, 1905.

Therefore the points O, P, and Q are inverse points. Similarly if the points P and Q be interchanged. Q. E. D.

SCHOLIUM. From the above it is evident that if the point O be fixed in position, and the points P, P', etc., be constrained to move in a circle through O, the point Q will describe a straight line, as AB.

THEOREM. *If O and Q be two adjacent vertices of a crossed parallelogram $OQRS$, and P be a point situated on a parallel to OS collinear with O and Q, then $OP \times OQ$ is constant, whether P be within or without the points O and Q.*



GIVEN: The crossed parallelogram $QROS$; $BA \parallel SO$; P on BA produced collinear with O and Q; AN and SM perpendicular to OP.

PROOF. $OP = ON \pm NP = \sqrt{(AO^2 - AN^2)} \pm \sqrt{(AP^2 - AN^2)}$.

$OQ = MQ \pm MO = \sqrt{(QS^2 - SM^2)} \mp \sqrt{(SO^2 - SM^2)}$.

Now $MS = \dfrac{OS}{AP} . AN$ and $QS = \dfrac{OS}{AP} . AO$.

Therefore $OP . OQ = \dfrac{OS}{AP} [AO^2 \mp AP^2]$ is a constant.

Hence the points O, P, and Q are inverse points. Q. E. D.

Similarly, is the points P', P'', etc., be taken.

Evidently then if the point O be fixed and P be constrained to move in a circle through O, the point Q will move in a straight line, as XY. The position of the line described by Q depends upon the position of the center of the circle described by the point P. It is perpendicular to the line connecting O to the center of the circle.